OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4732

Probability & Statistics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4732
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 26 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

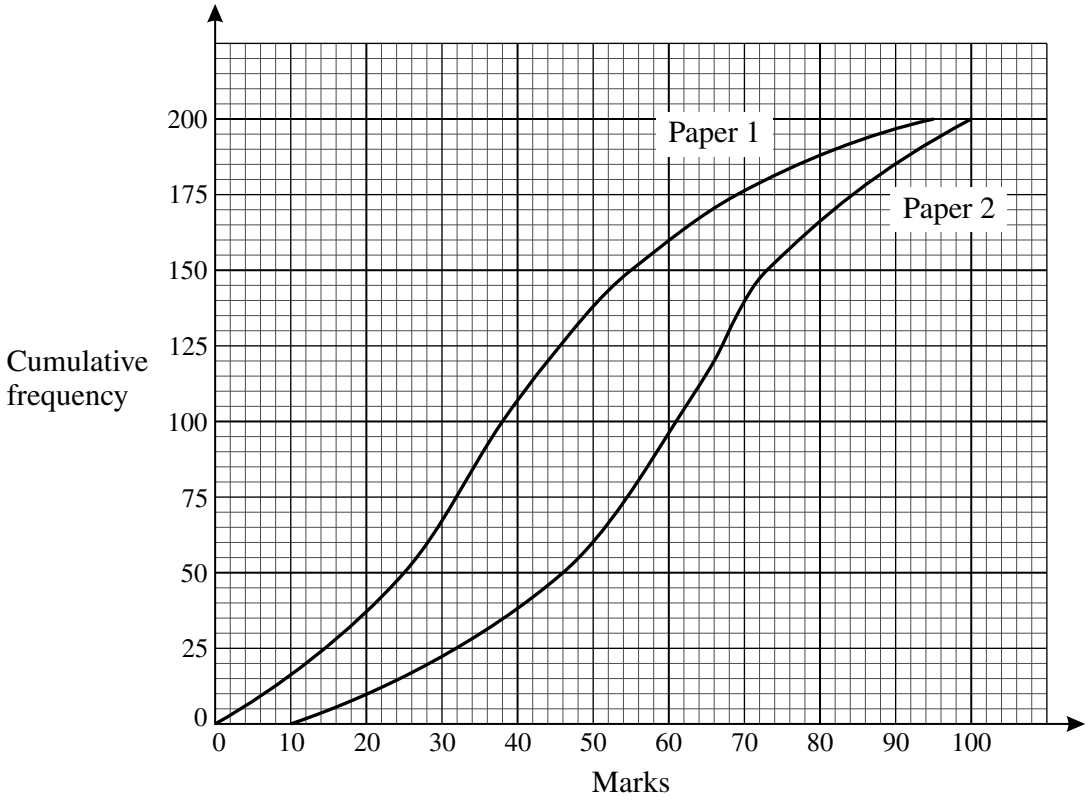
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 200 candidates took each of two examination papers. The diagram shows the cumulative frequency graphs for their marks.



(i) Estimate the median mark for each of the papers. **[2]**

(ii) State, with a reason, which of the two papers was the easier one. **[2]**

(iii) It is suggested that the marks on Paper 2 were less varied than those on Paper 1. Use interquartile ranges to comment on this suggestion. **[4]**

(iv) The minimum mark for grade A, the top grade, on Paper 1 was 10 marks lower than the minimum mark for grade A on Paper 2. Given that 25 candidates gained grade A in Paper 1, find the number of candidates who gained grade A in Paper 2. **[2]**

(v) The mean and standard deviation of the marks on Paper 1 were 36.5 and 28.2 respectively. Later, a marking error was discovered and it was decided to add 1 mark to each of the 200 marks on Paper 1. State the mean and standard deviation of the new marks on Paper 1. **[2]**

2 The random variable X has the distribution Geo(0.2). Find

(i) $P(X = 3)$, **[2]**

(ii) $P(3 \leqslant X \leqslant 5)$, **[3]**

(iii) $P(X > 4)$. **[3]**

Two independent values of X are found.

(iv) Find the probability that the total of these two values is 3. **[3]**

3 A firm wishes to assess whether there is a linear relationship between the annual amount spent on advertising, £x thousand, and the annual profit, £y thousand. A summary of the figures for 12 years is as follows.

$$n = 12 \quad \Sigma x = 86.6 \quad \Sigma y = 943.8 \quad \Sigma x^2 = 658.76 \quad \Sigma y^2 = 83\,663.00 \quad \Sigma xy = 7351.12$$

(i) Calculate the product moment correlation coefficient, showing that it is greater than 0.9. **[3]**

(ii) Comment briefly on this value in this context. **[1]**

(iii) A manager claims that this result shows that spending more money on advertising in the future will result in greater profits. Make two criticisms of this claim. **[2]**

(iv) Calculate the equation of the regression line of y on x. **[4]**

(v) Estimate the annual profit during a year when £7400 was spent on advertising. **[2]**

4 Jenny and Omar are each allowed two attempts at a high jump.

(i) The probability that Jenny will succeed on her first attempt is 0.6. If she fails on her first attempt, the probability that she will succeed on her second attempt is 0.7. Calculate the probability that Jenny will succeed. **[3]**

(ii) The probability that Omar will succeed on his first attempt is p. If he fails on his first attempt, the probability that he will succeed on his second attempt is also p. The probability that he succeeds is 0.51. Find p. **[4]**

5 30% of packets of Natural Crunch Crisps contain a free gift. Jan buys 5 packets each week.

(i) The number of free gifts that Jan receives in a week is denoted by X. Name a suitable probability distribution with which to model X, giving the value(s) of any parameter(s). State any assumption(s) necessary for the distribution to be a valid model. **[4]**

Assume now that your model is valid.

(ii) Find

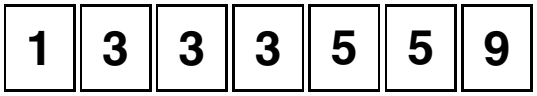
(a) $\mathrm{P}(X \leqslant 2)$, **[1]**

(b) $\mathrm{P}(X = 2)$. **[2]**

(iii) Find the probability that, in the next 7 weeks, there are exactly 3 weeks in which Jan receives exactly 2 free gifts. **[3]**

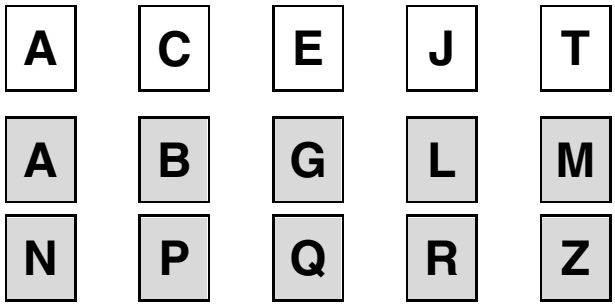
[Questions 6, 7 and 8 are printed overleaf.]

6 **(i)** The diagram shows 7 cards, each with a digit printed on it. The digits form a 7-digit number.



How many different 7-digit numbers can be formed using these cards? **[3]**

(ii) The diagram below shows 5 white cards and 10 grey cards, each with a letter printed on it.



From these cards, 3 white cards and 4 grey cards are selected at random **without** regard to order.

(a) How many selections of seven cards are possible? **[3]**

(b) Find the probability that the seven cards include exactly one card showing the letter A. **[4]**

7 The probability distribution of a discrete random variable, X, is shown below.

x	0	2
$P(X = x)$	a	$1 - a$

(i) Find $E(X)$ in terms of a. **[2]**

(ii) Show that $\text{Var}(X) = 4a(1 - a)$. **[3]**

8 Five dogs, A, B, C, D and E, took part in three races. The order in which they finished the first race was $ABCDE$.

(i) Spearman's rank correlation coefficient between the orders for the 5 dogs in the first two races was found to be -1. Write down the order in which the dogs finished the second race. **[1]**

(ii) Spearman's rank correlation coefficient between the orders for the 5 dogs in the first race and the third race was found to be 0.9.

(a) Show that, in the usual notation (as in the List of Formulae), $\Sigma d^2 = 2$. **[2]**

(b) Hence or otherwise find a possible order in which the dogs could have finished the third race. **[2]**

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1i	38 61	B1 B1 2	Reversed: B1B0	
ii	Paper 2 Higher median or curve is to right	B1 B1dep 2	Indep of reason or similar Higher average or mean or midpoint Paper 2: half ≤ 61, cf paper 1: half ≤ 38 Paper 1: more students scored lower marks (or lower than eg 40)	Ans "Paper 1", ignore reason: B0B0 unless reversed in (i) More scored higher mks Highest & lowest mks are higher For each cf, the corresponding mark is higher in p2. None get 0-10 Some get 100 Eg 25 scored > 69 in p1, cf 65 scored > 69 in p2 NOT Marks are higher NOT marks seem higher NOT everyone gets higher mks NOT Curve steeper Ignore irrelevant or incorrect SC: If reversed in (i): (ii) p1 because median higher B1B1ft
iii	55, 25 73, 46 Paper 1 IQR = 30 Paper 2 IQR = 27 Suggestion correct or p2 less varied	M1 A1 A1 B1f indep 4	M1 one pair of quartiles p2 more consistent or less spread out Allow "p2 has smaller range (or smaller variance") if IQRs found "It" is less varied: assume p2: B1	Allow 55±1, 25±1 Not necessarily subtracted 73±1, 46±1 30±1 27±1 p1 more varied or more spread out or less consistent Little difference or similarly varied NOT p2 IQR smaller than p1 unless also says less varied oe If quartiles found but not IQRs: max M1A0A0B1 If no quartiles calculated can still score B1 Steeper curve alone M0A0A0B0 If IQRs wrong, with p1 < p2, ft "suggestion wrong": B1f Ignore irrelevant or incorrect

iv	37 (± 3)	B2 2	B1 for 163 (± 3)	Not necessarily integer. B1 for 78-80 mks for min grade A on p2 SC: ans 105 – 110: B1 (from p1 10 mks hier instead of lower)
v	37.5 28.2	B1 B1 2	cao or sd the same	NOT eg 37.51 Ignore all working
Total		**12**		
2				SC:Consistent use of incorrect (1 – 0.2) score M-marks only SC:Consistent 0.8 insted of 0.2, no A-marks: max M0M2M2M2 "Consistent" means in every part attempted
2i	$0.8^2\times0.2$ $= \frac{16}{125}$ or 0.128	M1 A1 2		
ii	$0.8^2\times0.2 + 0.8^3\times0.2 + 0.8^4\times0.2$ $= \frac{976}{3125}$ or 0.312 (3 sfs)	M2 A1 3	1 term omitted or wrong or extra: M1	Using P($X\leq5$) & P($X\leq2$); three methods: $1 - 0.8^5 - (1-0.8^2)$ or 0.672 – 0.36: M2 Allow M1 for $1 - 0.8^5 - (1-0.8^3)$ or 0.672 – 0.488 or $1 - 0.8^4 - (1-0.8^2)$ or 0.5904 – 0.36 $0.8^2 - 0.8^5$: M2 Allow M1 for $0.8^3 - 0.8^5$ or $0.8^2 - 0.8^4$ $0.2+0.8\times0.2+0.8^2\times0.2+0.8^3\times0.2+0.8^4\times0.2 - (0.2+0.8\times0.2)$: M2 One term omitted or wrong or extra: M1 But NB If include $0.8^{-1}\times0.2$ in both P($X\leq5$) & P($X\leq2$), get correct ans but M1M0A0 M0 for eg $1 - 0.8^5 - 0.8^2$ or 0.672 – 0.64
iii	0.8^4 $= \frac{256}{625}$ or 0.4096 or 0.410 (3 sfs)	M2 A1 3	1-(0.2 + 0.8×0.2 + $0.8^2\times0.2$ + $0.8^3\times0.2$) 1 term omitted or wrong or extra: M1 $1 - 0.8^4$ or 0.590 M1 or 0.8^3 or 0.512 or 0.8^5 or 0.328: M1 Allow 0.41	$1 - (0.2 + 0.8\times0.2 + 0.8^2\times0.2 + 0.8^3\times0.2)$ M2 0.2×0.8^4 M0 $1 - 0.8^n$ ($n \neq 4$) M0

iv	$0.2 \times 0.8 \times 0.2$ $\times 2$ $= 0.064$ or $^{8}/_{125}$	M1 M1 A1 3	or $0.2 \times 0.8^{0} \times 0.8 \times 0.2$ or $0.2 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.2$	or 0.032 NOT $n \times 0.2^{2} \times 0.8$ except $n = 2$ Fully correct method except allow M0M1 for $(0.2+0.8\times0.2) \times 2$, must see method Attempt 0,3 and/or 3,0, as well as 2,1and/or 1,2; max M1M0A0 Careful: $0.2\times0.8\times0.2+0.2\times0.8^{-1}\times0.128 = 0.064$ M1M0A0 Careful: $0.8\times0.8\times0.2 \div 2 = 0.064$: (ie P($X = 3$) ÷ 2) M0M0A0
Total		**11**		
3i	$\frac{7351.12-\frac{86.6\times943.8}{12}}{\sqrt{(658.76-\frac{86.6^2}{12})(83663-\frac{943.8^2}{12})}}$ or $\frac{540.03}{\sqrt{33.80\times9433}}$ $= 0.9564\ldots$ or 0.956 or 0.96	M1 M1 A1 3	Must see at least 2 sfs	1st M1 for correct subst in any correct S formula 2nd M1 for all correct subst'n in any correct r formula 0.96 or correct better, no working: M1M1A1 eg 0.958 → 0.96 with correct working M1M1A0 without working: M0M0A0
ii	Strong (or high or good or close etc) relationship (or corr'n or link) between amount spent on advert & profit	B1 1	Allow Almost complete relationship or Very positive corr'n or Very reliable relationship or Near perfect relationship between spend on advert & profit oe, in context	Must state or imply "strong" or "good" or equiv & in context but NOT Strong *agreement* between etc NOT High spend on ads produces high profits NOT The more spent on adverts, the higher the profit NOT Positive corr'n between spend on ads & profits NOT There is a relationship between spend on ads & profit NOT There is a great relationship between etc NOT ans involving "proportion(al)" Ignore irrelevant or incorrect If incorrect r (< 0.9) in (i), no ft for ans "weak rel'nship" here; but correct ans here scores B1 even if inconsistent with their r

iii				Allow without context
	Relationship may not continue	B1	Can't extrapolate Any indication that pattern may not continue Must state or imply referring to future	Examples: Can't predict future; Things can change May be recession ahead; Economic situation may change Cost of advertising may increase If spend too much on ads, profit may be reduced as a result Advertising may not be as successful in the future Item may go out of fashion NOT Spending on adverts may not bring high profits NOT Spending more on adverts may not bring higher profits (Since these just restate the question) NOT More money spent on ads will not affect profit
	Corr'n not imply causation	B1 2	Increase in profit may not be due to increase in spend on advertising. Variables may be increasing separately	Both variables may be affected by a third Other factors may affect profits Advertising not the sole factor affecting profits Two different categories of reason needed, as given above. Two reasons which both fall under the same category: only B1 NOT Because corr'n not equal to 1
iv	$b = \frac{7351.12 - \frac{86.6 \times 943.8}{12}}{658.76 - \frac{86.6^2}{12}}$ = 15.9788 or 16.0 $y - \frac{943.8}{12} =$ "16.0"$(x - \frac{86.6}{12})$ $y = 16x - 37$ or better	M1 A1 M1 A1 4	or $\frac{S_{xy}}{Sxx}$ or $a = \frac{943.8}{12} -$ "16.0" $\times \frac{86.6}{12}$ ($y = 15.9788x - 36.664$)	ft values of S_{xy} & S_{xx} if clearly shown in (i) Coeffs not nec'y rounded, but would round to 16 & 37 These marks can be earned in (v) if not contradicted in (iv) If x on y line found: M-marks only ($x = 2.71 + 0.0572y$)
v	"16" × 7.4 – "37" 81400 to 81750	M1 A1f 2	 81.4 thousand to 81.7 thousand: M1A1 but 81.4 to 81.7 alone: M1A0	"16" × 7400 – "37": M0A0 ft their (iv)
Total		**12**		

4i	0.4×0.7 $0.6 + 0.4 \times 0.7$ $= 0.88$	M1 M1 A1 3	or 0.6 + prod of 2 probs Condone $0.6 \times 0.7 + 0.6 \times 0.3 + 0.4 \times 0.7$ or $0.6 \times 0.6 + 0.6 \times 0.4 + 0.4 \times 0.7$	1– prod of 2 P's or 0.4×0.3 $1 - 0.4 \times 0.3$
ii	$p + (1-p) \times p = 0.51$ or $2p - p^2 = 0.51$ $p^2 - 2p + 0.51 = 0$ $(p-0.3)(p-1.7) = 0$ or $p = \frac{2 \pm \sqrt{4-4\times0.51}}{2}$ oe $p = 0.3$	M1 A1 M1 A1 4	or $p^2 + p \times (1-p) + (1-p) \times p$ Correct QE = 0 Condone omission of "= 0" Correct method for their 3-term QE Not $p = 0.3$ or 1.7	Condone $p + p \times 1 - p$ M1, but $p + qp = 0.51$ M0 or $(1-p)^2 = 0.49$ M1A1 $1 - p = \pm 0.7$ M1 must have ± Correct ans from correct but reduced wking or T & I or verification or no wking: 4 mks Ans $p = 0.3$ or 1.7 from correct but reduced wking or T & I or no wking: M1M1M1A0 Ans $p = 0.3$ following correct wking except other solution incorrect: BOD 4 mks (eg $p = \frac{2 \pm \sqrt{4-4\times0.51}}{2}$ so $p = 0.3$ or -1.3 so $p = 0.3$: 4 mks)) $p = 0.3$ from wrong wking but correct verification: BOD 4 mks $p = 0.3$ from wrong wking alone: M0A0M0A0
Total		**7**		

5			Consistent use of $\frac{1}{3}$ or MR of 30% (eg 0.2): ("Consistent" as in Qu 2) (i) B1B0B1B1 (iia) B0 (iib) 0.7901–0.4609 or ${}^5C_2(\frac{2}{3})^3(\frac{1}{3})^2$ M1; = 0.329 (3 sf) A1 (iii) p = "0.3292" M1; ${}^7C_3(1 - \text{"0.3292"})^4(\text{"0.3292"})^3$ M1; = 0.253 (3 sf) A1 ie max 8/10	
5i	Binomial or B (5, 0.3) Prob of gift same for all pkts Whether pkt contains gift is indep of other pkts	B1 B1 B1 B1 4	 Prob of gift is constant or fixed or consistent or same oe Obtaining a gift is indep Each time receive a gift is indep Context needed for 3rd & 4th B-mks	Allow mis-spellings but NOT "Biometric" Condone B~(5, 0.3) or B(0.3, 5): B1B1 but B(X = 0.3, n = 5): B1B0 NOT: prob of success const; NOT prob stays same each go One box doesn't affect another. Pkts indep. Gifts indep She buys packets separately Prob of a gift is indep Prob of gift indep of one another & const: B1B1 NOT: Each week is indep NOT: Number of gifts received is indep NOT: Events indep If Geo(0.3) stated, can score max B0B0B1B1 If Geo(5, 0.3) stated, can score max B0B1B1B1
iia	0.8369	B1 1	or 0.837	
b	0.8369 – 0.5282 or ${}^5C_2(0.7)^3(0.3)^2$ = 0.3087 or 0.309 (3 sf)	M1 A1 2		
iii	p = "0.3087" ${}^7C_3(1 - \text{"0.3087"})^4(\text{"0.3087"})^3$ = 0.235 (3 sf)	M1 M1 A1 3	(iib) used in a calc'n eg "0.3087" × 3	or B(7, "0.3087") stated or 1 – "0.3087" used instead of "0.3087" n = 35 or 15: max M1M0A0
Total		**10**		

6i	$7! \div 3!$ $7! \div 2!$ $\div 2!$ $\div 3!$ $= 420$	M1 M1dep A1 3	But NOT ${}^7P_4$ or 7!/(7-4)! if seen	$\frac{7!}{3!+2!}$: M1M0 $\frac{7!}{3!\times n!}$ any n: M1M0
iia	${}^5C_3$ or ${}^{10}C_4$ seen ${}^5C_3 \times {}^{10}C_4$ $= 2100$	M1 M1 A1 3	or 10 or 210	$\frac{{}^5C_3 \times {}^{10}C_4}{\text{anything}}$ M1M1A0 ${}^5P_3 \times {}^{10}P_4$ or 60×5040 or 302400: SC B1
b	${}^4C_2 \times {}^9C_4$ or ${}^4C_3 \times {}^9C_3$ or 756 or 336 ${}^4C_2 \times {}^9C_4 + {}^4C_3 \times {}^9C_3$ or 1092 $\div 2100$ or $\div$ (iia) dep $\geq$ one M1 scored $= \frac{13}{25}$ or 0.52 "2100" – (${}^4C_3 \times {}^9C_4$ or ${}^4C_2 \times {}^9C_3$) or "2100" – (504 or 504) M1 "2100" – (${}^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_3$) M1 $\div$ "2100" or (iia) dep $\geq$ M1 M1	M1 M1 M1dep A1 4	$\frac{3}{5}$ or $\frac{4}{10}$ oe $\frac{3}{5} \times (1-\frac{4}{10})$ or $(1-\frac{3}{5}) \times \frac{4}{10}$ $\frac{3}{5} \times (1-\frac{4}{10}) + (1-\frac{3}{5}) \times \frac{4}{10}$ $= \frac{13}{25}$ $\frac{3}{5}$ or $\frac{4}{10}$ oe M1 $\frac{3}{5}+\frac{4}{10}-\frac{3}{5}\times\frac{4}{10}$ M1 $\frac{3}{5}+\frac{4}{10}-\frac{3}{5}\times\frac{4}{10}-\frac{3}{5}\times\frac{4}{10}$ M1 $= \frac{13}{25}$ A1	Not from incorrect wking SC $\frac{1}{5}\times\frac{9}{10}$ or $\frac{4}{5}\times\frac{1}{10}$ M1 $\frac{1}{5}\times\frac{9}{10} + \frac{4}{5}\times\frac{1}{10}$ M1 (= $\frac{13}{50}$ A0) Not from incorrect wking ie P(WA or GA or both) Must be correct figures ie P(WA or GA but not both) Must be correct figures SC : ${}^4P_2 \times {}^9P_4 + {}^4P_3 \times {}^9P_3$: M1 $\div$ (iia) M1dep Careful: 336 or 756 can be obtained by incorrect methods.
Total		**10**		

7i	$(0\times a) + 2\times(1-a)$ $= 2 - 2a$ or $2(1-a)$ oe	M1 A1 2	or $2(1-a)$ Not ISW	Condone $2 \times 1 - a$ NB $2\times(1-a) \div 2$: M0A0 Eg E(X) = $2 - 2a$; $2 - 2a = 1$; $a = 0.5$: M1A0
ii	$(0\times a) + 2^2\times(1-a)$ $-$ "$(2-2a)$"^{2} $= 4 - 4a - 4 + 8a - 4a^2$ $= 4a - 4a^2$ $(= 4a(1-a))$ **AG** $\begin{array}{\|c\|c\|}\hline -2+2a & 2a \\ \hline a & 1-a \\ \hline\end{array}$ M1 Var(X) $= a(-2+2a)^2 + 4a^2(1-a)$ M1 $4a^3 - 8a^2 + 4a + 4a^2 - 4a^3$ $4a - 4a^2$ A1	M1 M1 A1 3	or $4 - 4a$ oe $-$ (i)^2 dep contains a; ISW; Indep mk or $4(1-a) - 4(1-a)^2$ $4(1-a)(1-(1-a))$ Correct table oe	Condone $2^2 \times 1 - a$ $4 - 4a - 4 \pm 8a \pm 4a^2$ or $4 - 4a - 4 \pm 4a^2$ or equiv M1M1A0 $4 - 4a - 2(1-a)^2$ M1M1A0 Must see this line, correctly obtained Careful: $4 - 4a - (2-2a)^2 = 4 - 4a - (4 - 4a^2) = -4a + 4a^2 = 4a(1-a)$ M1M1A0 only
Total		**5**		
8i	EDCBA	B1 1	A 5 B 4 C 3 D 2 E 1	NOT just 5, 4, 3, 2, 1
iia	$1 - \frac{6\Sigma d^2}{5(5^2-1)} = 0.9$ $1 - \frac{6\times\Sigma d^2}{5\times24} = 0.9$ or $0.1 = \frac{6\times\Sigma d^2}{5\times24}$ ($\Sigma d^2 = 2$ **AG**)	M1 A1 2	 One correct step or better & nothing incorrect for A1	$1 - \frac{6\times2}{5(5^2-1)}$ $= 1 - \frac{6\times2}{5\times24}$ or $1 - \frac{12}{5\times(5^2-1)}$ One correct step or better & nothing incorrect for A1 (= 0.9 **AG**)
b	d^2: 0, 0, 0, 1, 1 any order BACDE or similar	M1 A1 2	or d: 0, 0, 0, 1, -1 any order Any two adjacent dogs interchanged	May not be seen If **clearly** comparing second race with third; DECBA or similar: B1, but must be clear
Total		**5**		

Total 72 marks